New Measurements of Turbulent Shear Stresses in Hypersonic Boundary Layers

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Measurements in the turbulent hypersonic boundary layer over a slender sharp cone at $M_e = 7.1$ have been made with X-array hot wires for both cold ($T_w/T_{0e} = 0.41$) and adiabatic ($T_w/T_{0e} = 0.8$) wall conditions. In both cases, the hot-wire signals have been resolved to provide the entropy and vorticity fluctuation intensities and the double correlation terms appearing in the compressible shear stress expression. The results, which are consistent with previous findings at Mach 9.4, indicate a significant reduction in the "Reynolds stress" component $\bar{\rho}\,\overline{u'v'}$ with increasing M_e and show further that the effects of Mach number dominate those of heat transfer.

Nomenclature

$A_i\,(i=1\text{-}3)$	= constants arising from mode analysis, Eqs. (2-7)
$B_i\,(i=1\text{-}3)$	= constants arising from mode analysis, Eqs. (2-7)
$C_i\,(i=1,2)$	= constants arising from mode analysis, Eqs. (2-7)
$D_i\,(I=2,3)$	= constants arising from mode analysis, Eqs. (2-7)
e'	= normalized hot-wire voltage fluctuations
e_σ	= sensitivity coefficient to entropy fluctuations
e_τ	= sensitivity coefficient to vorticity fluctuations
I	= intercept derived from mode correlation analysis and defined in Eq. (4)
i	= hot-wire current
M	= Mach number
p	= pressure
Re_θ	= Reynolds number based momentum thickness
R_{ij}	= cross-correlation coefficient of i and j fluctuations
S	= slope derived from mode correlation analysis and defined in Eq. (4)
T_0	= stagnation temperature
u	= streamwise velocity component
v	= normal velocity component
X	= wire overheat parameter
y	= distance normal to surface
Y	= nondimensional hot-wire signal
Z	= nondimensional correlation signal
δ	= boundary-layer thickness
φ	= angle between hot-wire and mean velocity vector
ρ	= density
θ	= static temperature
$\langle(\ )\rangle$	= rms fluctuation
Subscripts	
e	= boundary-layer edge condition
0	= stagnation condition
w	= wall condition
Superscript	
$\overline{(\ )}$	= mean value
$(\ )'$	= fluctuating value

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Introduction

FOR the steady, two-dimensional turbulent boundary layer, contributions from flow fluctuations yield the following expressions for the turbulent shear stress:[1]

$$\frac{\partial}{\partial y}\left(\bar{\rho}\ \overline{u'v}+\bar{u}\ \overline{\rho'v'}+\bar{v}\ \overline{\rho'u'}+\overline{\rho'u'v'}\right)$$

In the incompressible case, the latter three terms vanish and the previous expression reduces to the "Reynolds Stress" $\bar{\rho}\ \overline{u'v'}$. For the compressible case, the last two terms are usually assumed to be negligible while it is common practice to mask the second term by reformulating the equations of motion in mass-averaged coordinates,[2] so that again the turbulent shear can be represented solely by the Reynolds stress term.

Although measurements of the Reynolds stress component $\bar{\rho}\,\overline{u'v'}$ extend over the Mach number range from 0-9.4, the available data are limited in number and are unable to clearly define the effects of heat transfer and compressibility. Specifically, the Mach 3 adiabatic wall measurements of Yanta and Lee[3] and of Johnson and Rose,[4] when normalized by the freestream dynamic pressure, show only a slight decrease below the incompressible results of Klebanoff.[5] On the other hand, the cold wall ($T_w/T_{0e} = 0.38$) measurements of Demetriades and Laderman[6] at Mach 9.4 indicate a hundred-fold reduction compared to the low-speed data. This large decrease has been cause for concern since indirect methods for extracting the Reynolds stress from measured mean flow profiles (e.g., see Ref. 7) imply a more gradual decline with Mach number. More recently, Mikulla and Horstman[8] reported cold wall shear stress measurements at Mach 6.85. These are generally consistent with the previous data but are still larger than could be expected on the basis of Ref. 6. Clearly, additional evidence is required to establish a reliable data trend and to resolve the discrepancy between the direct stress measurements and inferences from mean flow data.

This paper describes the results of new measurements of turbulent shear stresses at Mach 7.1 for both cold wall and adiabatic conditions. These results reinforce the findings of Ref. 6 and permit assessment of the effects of wall temperature and Mach number. In addition, the measurements provide an evaluation of the relative magnitude of the three double correlations in the turbulent shear stress expression.

The Experiment

The tests were performed in the Arnold Engineering Development Center Tunnel B, which was operated at a supply pressure P_0 and supply temperature T_0 of 500 psia and 1360° R, respectively, corresponding to a freestream Mach number of 8 and freestream Reynolds number of 1.8×10^5/in. The model used in the experiments was a 50-in. long, 4° half-angle, water-cooled, sharp-tip cone. The model was instrumented with four chromel-alumel thermocouples located flush with the cone surface at the $x=14.8$, 23.8, 34.8, and 49.7 in. stations. Four static pressure ports were located 90° apart at the $x=38.7$ in. station. The nominal edge Mach number M_e for both cold and adiabatic wall tests was 7.1. For the cold wall tests, the model was cooled to about 560° R, or a wall to total temperature ratio, T_w/T_{0e}, of 0.41. while for the adiabatic tests the wall temperature was 1080° R, corresponding to $T_w/T_{0e}=0.80$.

Surveys were carried out at approximately 48 in. from the cone tip where the boundary layer was fully turbulent. For the cold wall tests, the boundary-layer thickness δ was 0.35 in. and the Reynolds number, based on momentum thickness $R_{e\theta}$, was 2000, while for the adiabatic condition $\delta=0.30$ in. and $R_{e\theta}=1400$. The mean flow properties in the boundary layer were measured using pitot-pressure and total-temperature probes similar to those described in Ref. 9. The recovery factor characteristics of the total-temperature probe were determined by calibration in the tunnel freestream and it was assumed that the static pressure was constant across the boundary layer. Fluctuation measurements were made using X-array probes operated in the constant current mode with hot-wire diameters of 20 μ in. and aspect ratios of about 170. Construction details of the hot-wire probes are given in Refs. 10 and 11 while a description of the procedure used to determine the effective angle of inclination of the wire elements to the flow is discussed in Ref. 11. The hot wires were "oven-calibrated" prior to the tests to determine their temperature coefficient of resistivity and "flow-calibrated" during the tests to derive the heat transfer characteristics needed to calculate the sensitivity coefficients.[12-14]

Method of Analysis

The Hot-Wire Equations

The theory of the X-probe response in compressible flow, including allowances for pressure fluctuations p', has been described in detail.[12] Since it is known that p' fluctuations become significant above Mach 5, it would seem necessary to account for their effects in the present experiment. However, in order to resolve the hot-wire signal into its various contributions in the presence of pressure fluctuations, it is also necessary to make specific assumptions concerning the nature of the sound field. On the other hand, the results of Ref. 13 indicate that excluding pressure fluctuations (i.e., setting $p'=0$) has only a small effect on the velocity and temperature fluctuations and it was felt that this assumption would also yield a reasonable estimate of the shear stress.

For the case $p'=0$, the instantaneous response of hot wires 1 and 2 of the X probe is given by

$$e'_i(t)=e_{\tau i}\left[\frac{u'(t)}{\bar{u}}+\frac{v'(t)}{\bar{u}}\cot\varphi_i\right]+e_{oi}\frac{\theta'(t)}{\bar{\theta}};\ i=1,\,2 \tag{1}$$

As usual, a third relation is supplied by the correlation $e'_1(t)$ $e'_2(t)$ of the hot-wire signals. After some algebra, these expressions can be put in the familiar form[6,14,15] involving the modal variables X and Y

$$Y_1{}^2\equiv\overline{[e'_1/e_{o1}]^2}=A_1X_1{}^2+A_2X_1+A_3 \tag{2}$$

$$Y_2{}^2\equiv\overline{[e'_2/e_{o2}]^2}=B_1X_2{}^2+B_2X_2+B_3 \tag{3}$$

$$Z\equiv\overline{e'_1e'_2}/e_{o1}e_{o2}=X_1(X_2C_1+C_2)+X_2D_2+D_3 \tag{4}$$

where:

$$X_1\equiv e_{\tau 1}/e_{o1};\ X_2\equiv e_{\tau 2}/e_{o2}$$

$$A_1=\frac{\overline{u'^2}}{\bar{u}^2}+\cot^2\varphi_1\frac{\overline{v'^2}}{\bar{u}^2}+2\cot\varphi_1\frac{\overline{u'v'}}{\bar{u}^2} \tag{5a}$$

$$B_1=\frac{\overline{u'^2}}{\bar{u}^2}+\cot^2\varphi_2\frac{\overline{v'^2}}{\bar{u}^2}+2\cot\varphi_2\frac{\overline{u'v'}}{\bar{u}^2} \tag{5b}$$

$$C_1=\frac{\overline{u'^2}}{\bar{u}^2}+\cot\varphi_1\cot\varphi_2\frac{\overline{v'^2}}{\bar{u}^2}+(\cot\varphi_1+\cot\varphi_2)\frac{\overline{u'v'}}{\bar{u}^2} \tag{5c}$$

$$A_2=C_2=\frac{\overline{u'\theta'}}{\bar{u}\bar{\theta}}+\cot\varphi_1\frac{\overline{v'\theta'}}{\bar{u}\bar{\theta}} \tag{6a}$$

$$B_2=D_2=\frac{\overline{u'\theta'}}{\bar{u}\bar{\theta}}+\cot\varphi_2\frac{\overline{v'\theta'}}{\bar{u}\bar{\theta}} \tag{6b}$$

$$A_3=B_3=D_3=\overline{\theta'^2}/\bar{\theta}^2 \tag{7}$$

Equations (2) and (3) can be solved for the A_i's and B_i's by operating each hot wire at a minimum of three overheat currents as usual. Similarly, Eq. (4) can be solved for C_i and D_i using the method of Ref. 15. Briefly, this involved holding X_2 constant and solving the Z vs X_1 relation for the slope $s=x_2C_1+C_2$ and intercept $I=X_2D_2+D_3$. This is repeated for several values of X_2 and the X_2-S and X_2-I data curve fit to yield the coefficients C_1,C_2,D_2, and D_3. To improve the accuracy of these operations, the first wire was operated at a total of 14 overheat currents i_1, while wire No. 2 was operated at 5 currents i_2, each of which was held constant while i_1 was varied three times (only twice for the fifth value of i_2). The coefficients of the resulting overdetermined set of equations were determined by the method of least squares.

It should be noted that the results presented here were obtained without carrying out the lengthy response restoration

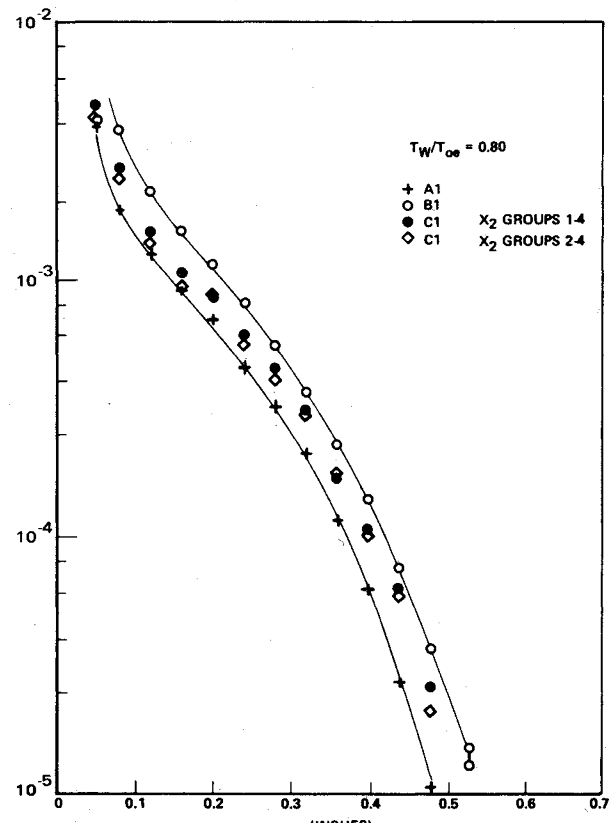

Fig. 1 Plot of mode coefficients A_1, B_1, and C_1 vs y position for adiabatic case, $T_w/T_{0e}=0.8$.

procedure[13] needed to correct for the limited frequency response of the hot-wire instrumentation. This problem has been investigated for a similar boundary-layer flow in Ref. 9 where it was shown that the effect of omitting the response restoration is most severe immediately adjacent to the wall, where the fluctuations are underestimated by a factor of 1.5-2, and gradually vanishes as the boundary-layer edge is approached. Since shear stress is proportional to the square of the fluctuations, the possible (but not certain) correction to the stress term near the wall ranges from 2-4. In the center of the boundary layer, where most of the measurements were made, the correction is proportionately smaller, or about a factor of 1.5-2. It should be further noted that the previous equations are the result of a linearized analysis based on small disturbances in the flow variables. As shown later, the peak temperature fluctuations for the adiabatic case approach 30% so that the application of the linear theory to the present data can be questioned. The results, in fact, indicate the need to modify the basic modal analysis to accommodate large fluctuation intensities and to examine the possibility of coupling between various turbulent modes. These questions, while important, are believed to have only a minor effect on the fluctuations computed here and their resolution is deferred to a later time.

Effect of Errors

At this point, it should be emphasized that errors and uncertainties inherent in the measurements and introduced by the several least squares procedures will reflect in the various coefficients. To anticipate the effect of uncertainties on the final results, Eqs. (5a-5c) have been solved to obtain explicit relations for the fluctuation terms. Since cot $\varphi_1 \approx 1$ and cot $\varphi_2 \approx -1$, these expressions can be approximated by

$$(\overline{u'^2}/\bar{u}^2) = (A_1 + B_1 + 2C_1)/4$$

$$(\overline{v'^2}/\bar{u}^2) = (A_1 + B_1 - 2C_1)/4$$

$$(\overline{u'v'}/\bar{u}^2) = (A_1 - B_1 - C_1(\epsilon))/4$$

where $\epsilon = \cot^2\varphi_2 - \cot^2\varphi_1$. A plot of the coefficients A_1, B_1, C_1 vs position y for the adiabatic case is shown in Fig. 1. With the exception of $y = 0.048$ in., the A_1 and B_1 data appear to be well behaved and B_1 is approximately twice A_1. Two plots of C_1 have been included, one obtained using the first four X_2 groups in the $S-X_2$ curve fit procedure, the other with the first X_2 group omitted. This was done to show that C_1 is relatively insensitive to slight variations in the data input (other selections of X_2 groups were also made with similar effect on C_1). C_1 is seen to be similar to magnitude to A_1 and B_1 and, more specifically, $A_1 < C_1 < B_1$.

It is clear, therefore, that errors or uncertainties in $\overline{u'^2}/\bar{u}^2$ will be similar in magnitude to those in the quantities A_1, B_1, and C_1. In addition, while $\overline{u'v'}/\bar{u}^2$ is proportional to $A_1 - B_1$, the term B_1 is approximately twice A_1 so that errors in $\overline{u'v'}$ should not be excessive. The contribution of C_1 to $\overline{u'v'}/\bar{u}^2$ is an order of magnitude smaller than that of either A_1 or B_1 and therefore should have negligible effect on the correlation terms. The $\overline{v'^2}/\bar{u}^2$ term, however, is found by subtracting two terms of similar magnitude. Thus, even small errors in the mode coefficients can produce large uncertainties in $\overline{v'^2}/\bar{u}^2$ and can, in fact, cause $\overline{v'^2}/\bar{u}^2$ to change sign. For example, a difference of 5-10% in C_1, resulting from a different curve fit used in the mode correlation analysis, causes a change in the sign of $\overline{v'^2}/\bar{u}^2$. It is concluded, therefore, that the fluctuations $\overline{u'^2}/\bar{u}^2$ and $\overline{u'v'}/\bar{u}^2$ are relatively insensitive to small errors in the mode coefficients while the accuracy with which $\overline{v'^2}/\bar{u}^2$ can be determined is inherently poor. It must be realized, however, that this is a consequence of the procedure used to find $\overline{v'^2}/\bar{u}^2$ and does not reflect on the quality of the data.

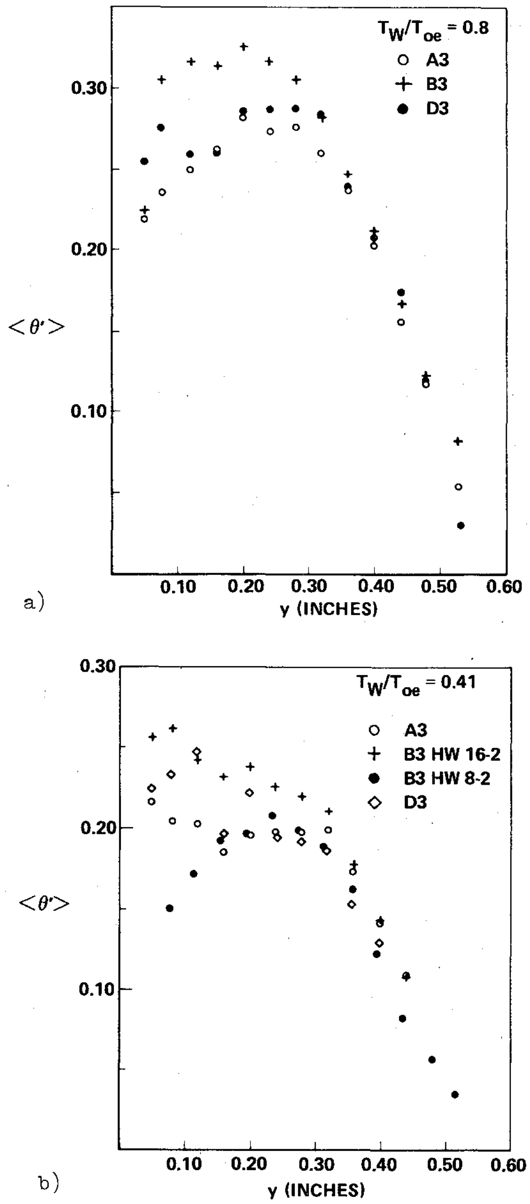

Fig. 2 Root-mean-square static temperature fluctuations: a) adiabatic wall, $T_w/T_{0e} = 0.8$; b) cold wall, $T_w/T_{0e} = 0.41$.

A similar evaluation can be made concerning the velocity-temperature correlations. From Eqs. (6a) and (6b) it is seen that the $\overline{u'\theta}/\bar{u}\bar{\theta}$ term is found from the sum of the mode coefficients A_2 (or C_2) and B_2 (or D_2) while $\overline{v'\theta'}/\bar{u}\bar{\theta}$ is determined from the difference of the terms. Consequently, the $\overline{v'\theta'}/\bar{u}\bar{\theta}$ term is considerably more sensitive to errors in the mode coefficients.

Results

The rms temperature fluctuations $\langle\theta'\rangle$ for the cold wall and adiabatic conditions are shown in Fig. 2. Except for positions very close to the wall ($y/\delta < 0.16$), in both cases the results obtained from the coefficients A_3, B_3, and D_3 are essentially the same (the similarity in the results provided independently by both hot-wire elements of the X-probe helps to establish confidence in the validity of the remaining fluctuation terms extracted from the mode analysis). While the adiabatic results, Fig. 2a, are in good agreement with the temperature fluctuations found in a similar cone study,[9] the present results for the cold wall case are larger than expected on the basis of previous cold wall measurements. As shown in Fig. 2b, the cold wall results were verified by independent

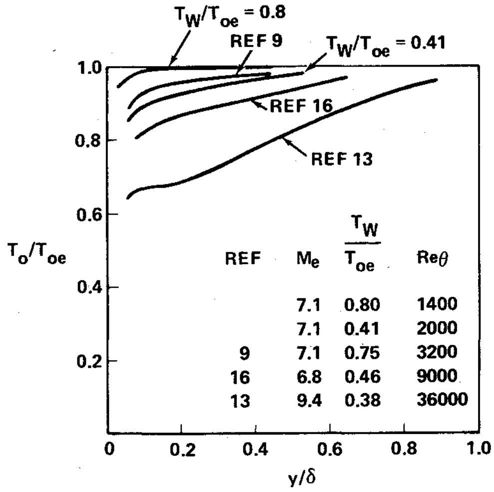

Fig. 3 Mean total temperature profiles.

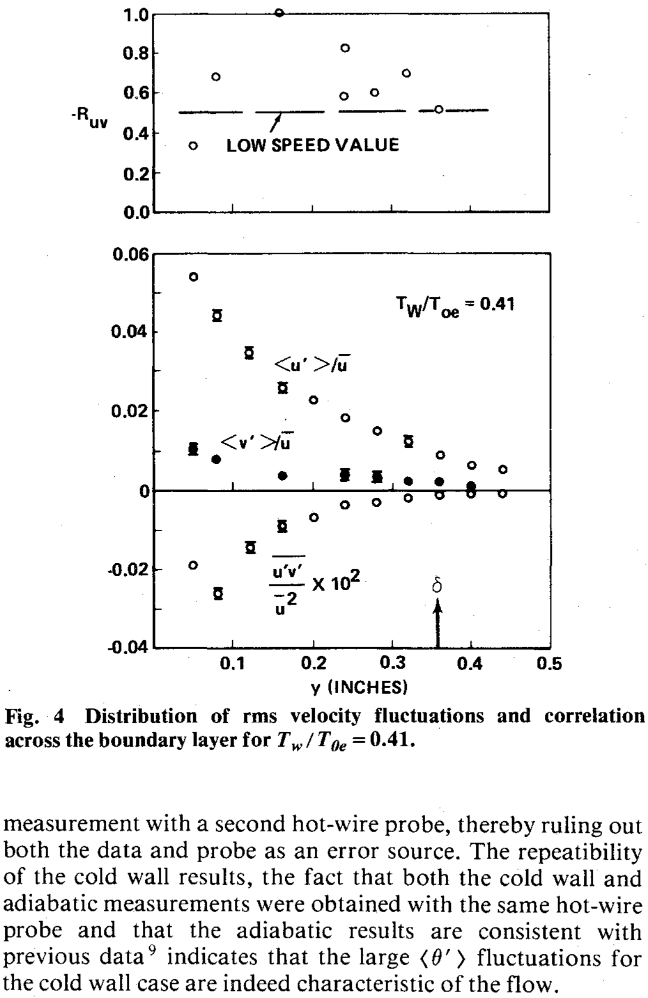

Fig. 4 Distribution of rms velocity fluctuations and correlation across the boundary layer for $T_w/T_{0e} = 0.41$.

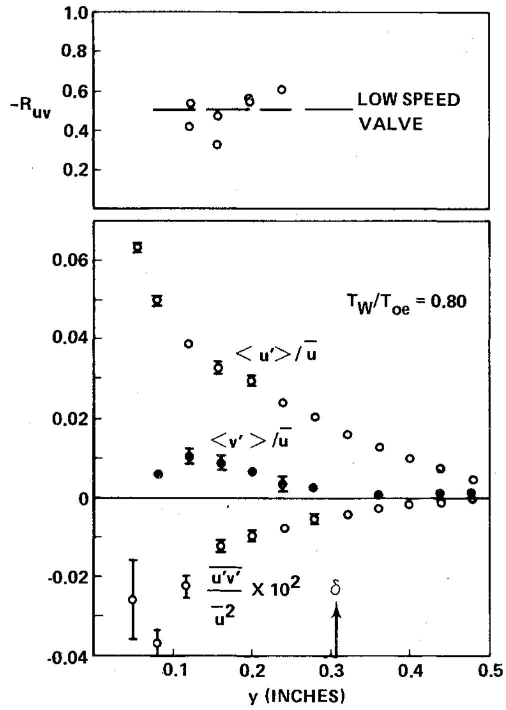

Fig. 5 Distribution of rms velocity fluctuations and correlation across the boundary layer for $T_w/T_{0e} = 0.8$.

measurement with a second hot-wire probe, thereby ruling out both the data and probe as an error source. The repeatibility of the cold wall results, the fact that both the cold wall and adiabatic measurements were obtained with the same hot-wire probe and that the adiabatic results are consistent with previous data[9] indicates that the large $\langle\theta'\rangle$ fluctuations for the cold wall case are indeed characteristic of the flow.

A possible explanation for the cold wall results is provided by examination of the total temperature profile which is shown in Fig. 3 where it is compared to T_0 profiles obtained from other experiments with similar T_w/T_{0e}. In spite of the relatively low value of T_w, the T_0 profile differs only slightly from that for an adiabatic wall. However, when compared to the data of Refs. 13 and 16, it appears that as Re_θ increases the departure from an adiabatic-like profile becomes more prominent, i.e., there is a decrease in the overall total temperature level throughout the boundary layer. This suggests that although transition is complete and the boundary layer is fully turbulent, because of the low Re_θ, it is not completely equilibrated. Since the cold wall T_0 profile is characteristic of the adiabatic case, it is not surprising that the corresponding $\langle\theta'\rangle$ fluctuations behave in a similar fashion. The effect of Re_θ on the velocity fluctuations is discussed later.

The velocity fluctuations for the cold wall and adiabatic conditions are plotted vs y in Figs. 4 and 5, respectively. Both figures include the rms fluctuations $\langle u'\rangle/\bar{u}$ and $\langle v'\rangle/\bar{u}$, the correlation $\overline{u'v'}/\bar{u}^2$, and the correlation coefficient R_{uv}. The spread in the results is the consequence of using different X_2 groups in the mode correlation analysis to calculate C_l. It is immediately obvious that both $\langle u'\rangle/\bar{u}$ and $\overline{u'v'}/\bar{u}^2$ are almost completely independent of the variations in the curve fitting process. However, as mentioned earlier, $\overline{v'^2}$ is particularly sensitive to uncertainties in C_l, and in cases where $2C_l > B_l + A_l$ this term was negative so that the corresponding $\langle v'\rangle/\bar{u}$ could not be evaluated nor included in Figs. 4 and 5. The values $\langle v'\rangle/\bar{u}$ that are plotted, however, are sufficient to indicate both the magnitude and trend of this term through the boundary layer. A partial check on the consistency of the results shown is provided by the correlation coefficient, defined as

$$R_{uv} = \overline{u'v'}/\langle u'\rangle\langle v'\rangle$$

While $\overline{u'v'}$ is an independent quantity, R_{uv} is not and, in fact, in the present case reflects the same uncertainties associated with $\langle v'\rangle$. However, on purely physical grounds it is known that R_{uv} is restricted to values between 0 and -1 and, based on previous measurements[4-6] ranging from $M = 0$-9.4, is expected to be about -0.5. A plot of R_{uv} has been included in the upper portion of Figs. 4 and 5. It is seen that the values of R_{uv} scatter about -0.5 and in most cases are sufficiently close to -0.5 to establish a fair degree of confidence in the present results.

A comparison of Figs. 4 and 5 leads to the following major findings. First, the velocity fluctuations for the cold wall case are smaller than those for the adiabatic condition, with $\langle u'\rangle/\bar{u}'$ decreasing approximately 20% and $\overline{u'v'}/\bar{u}^2$ reduced by about 40-50%. Second, $\langle v'\rangle/\bar{u}$ is 4-6 times less than $\langle u'\rangle/\bar{u}$, implying a greater anisotropy than at low speeds ($M \sim 0$) where $\langle v'\rangle/\bar{u} \approx 0.7\langle u'\rangle/\bar{u}$. Finally $\overline{u'v'}/\bar{u}^2$ appears to decrease near the wall, a characteristic common to all high

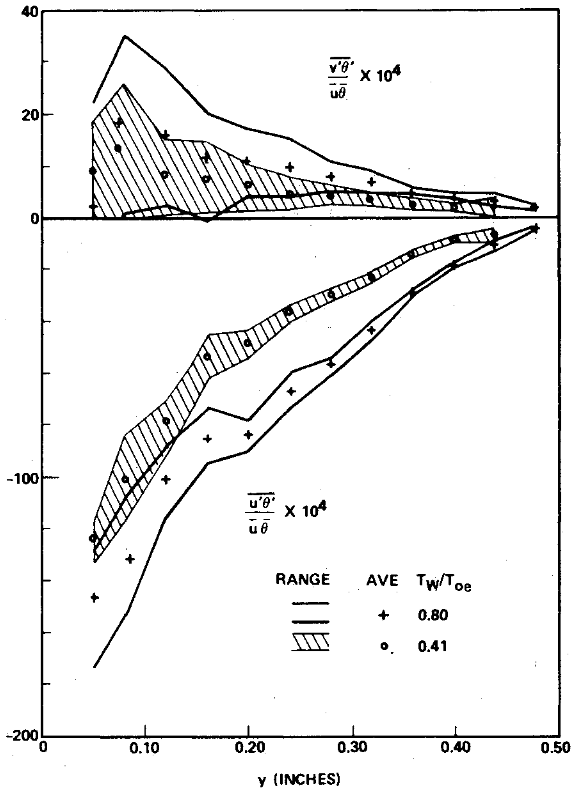

Fig. 6 Velocity temperature correlations. Shown are the range of maximum and minimum values while symbols denote average values.

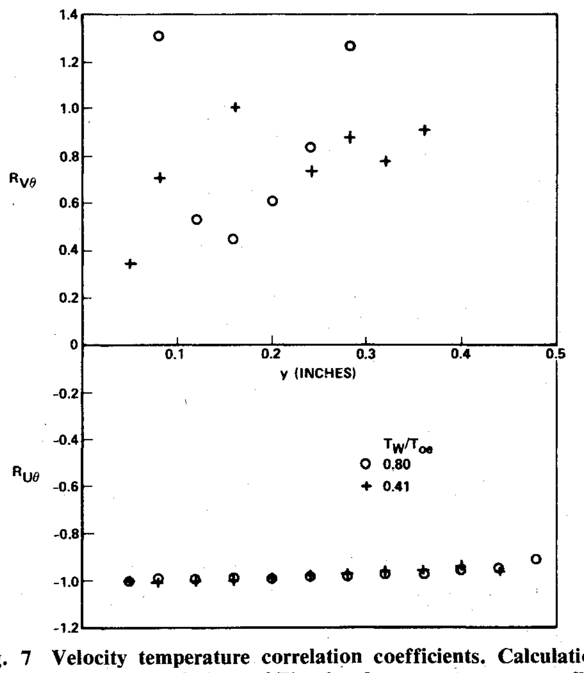

Fig. 7 Velocity temperature correlation coefficients. Calculations based on average correlations of Fig. 6 and average temperature fluctuations of Fig. 2.

speed measurements, including those made with the laser doppler velocimeter as well as the hot-wire anemometer.[4]

In view of the apparent effect of Re_θ on the cold wall measurements of T_0 and $\langle\theta'\rangle$, the question arises whether a similar effect can be expected on $\langle u'\rangle/\bar{u}$ and $\overline{u'v'}/\bar{u}^2$. A recent comprehensive review[17] of turbulence in compressible boundary layers demonstrates that the velocity fluctuations exhibit a much smaller sensitivity to wall temperature than do the temperature fluctuations and this same insensitivity should extend to the shear stress $\overline{u'v'}$. The results of Fig. 4, therefore, are considered representative of a cold wall.

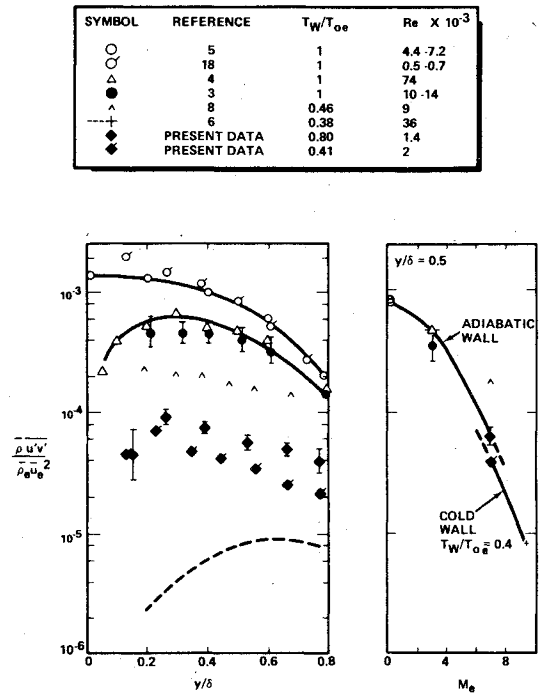

Fig. 8 Variation of nondimensional Reynolds shear stress $\bar{\rho}\,\overline{u'v'}/\rho_e u_e^2$ across the boundary layer and comparison with other experiments showing effect of Mach number and wall temperature.

A plot of the velocity-temperature correlations vs y position for both cold and adiabatic wall conditions is shown in Fig. 6. Calculations were carried out for the four possible combinations of the mode coefficients A_2, B_2, C_2, D_2 so that the data shown in Fig. 6 includes, therefore, the range (maximum and minimum) and average value of the correlation terms. As indicated previously, the accuracy of $\overline{v'\theta'}/\bar{u}\bar{\theta}$, which is obtained by subtracting two large terms of similar magnitude, is poor. However, the average value of both $\overline{v'\theta'}/\bar{u}\bar{\theta}$ and $\overline{u'\theta'}/\bar{u}\bar{\theta}$ shows a consistent and uniform behavior through the boundary layer with both correlations varying from near zero at the edge to a maximum value near the wall. It is of interest to note that both $\overline{u'\theta'}/\bar{u}\bar{\theta}$ and $\overline{v'\theta'}/\bar{u}\bar{\theta}$ for the adiabatic case exceed their corresponding values for the cold wall condition, reflecting the larger velocity and temperature fluctuations associated with the former. Of greater interest, it is seen that $\overline{v'\theta'}/\bar{u}\bar{\theta}$ is about an order of magnitude larger than $\overline{u'v'}/\bar{u}^2$. Since in the absence of pressure fluctuations $\overline{v'\theta'}/\bar{u}\bar{\theta} = -\overline{v'\rho'}/\bar{u}\bar{\rho}$, then the stress component $\bar{u}\,\overline{v'\rho'}$ is likewise ten times larger than the Reynolds stress $\bar{\rho}\,\overline{u'v'}$. This agrees with the findings of Ref. 6 and shows that the $\bar{u}\,\overline{v'\rho'}$ contribution dominates the turbulence shear stress in the hypersonic regime.

The correlation coefficients $R_{u\theta}$ and $R_{v\theta}$ for both wall conditions are plotted vs y position in Fig. 7. These calculations are based on the average values of $\overline{u'\theta'}/\bar{u}\bar{\theta}$ and $\overline{v'\theta'}/\bar{u}\bar{\theta}$ shown in Fig. 6 and the average temperature fluctuation determined from the A_3, B_3, and D_3 calculations. $R_{u\theta}$ is seen to be independent of wall temperature and nearly equal to -1.0 across the boundary layer, in agreement with previous hypersonic results.[9,13] Although the scatter in $R_{v\theta}$ is much larger, reflecting the uncertainties in both $\overline{v'\theta'}/\bar{u}\bar{\theta}$ and $\langle v'\rangle/\bar{u}$, this term also appears to be independent of wall temperature. In addition, $R_{v\theta}$ is positive and ranges from 0.3-0.9 through the boundary layer.

A comparison of the present shear stress measurements with those of other investigations[3-6,8,18] is presented in Fig. 8 where the Reynolds stress component arising from the velocity cross-correlation, normalized by the edge value of dynamic pressure, has been plotted as a function of y/δ. Figure 8 indicates that there is now sufficient data to delineate the effects of both wall temperature and Mach number M_e on the stress term $\bar{\rho}\,\overline{u'v'}/\bar{\rho}_e\bar{u}_e^2$. The insert on the right-hand side of Fig. 8, where the magnitude of this term at $y/\delta=0.5$ has been plotted against M_e, shows that the normalized shear stress decreases with decreasing wall temperature (which is directly a consequence of the corresponding decrease in the component velocity fluctuations) and, quite drastically, with increasing Mach number. (Note that even if the response corrections mentioned earlier are applied, the present results remain considerably below the others shown in Fig. 8 and are still 2.5-3 times smaller than those of Ref. 8). In fact, the present results confirm the trend revealed in Ref. 6 of a precipitous drop in $\bar{\rho}\,\overline{u'v'}/\bar{\rho}_e\bar{u}_e^2$ as M_e increases from 0-9.4. The causes contributing to the large reduction in magnitude can be identified by rewriting the stress term as:

$$\frac{\bar{\rho}}{\bar{\rho}_e}\frac{\overline{u'v'}}{\bar{u}_e{}^2}=R_{uv}\frac{\langle u'\rangle}{\bar{u}_e}\frac{\langle v'\rangle}{\bar{u}_e}\frac{\bar{\rho}}{\bar{\rho}_e}$$

In this expression $\langle u'\rangle/\bar{u}_e$ is the "vorticity" contribution to the stress, $\langle v'\rangle/\bar{u}_e$ represents the "isotropy" contribution, $\bar{\rho}/\bar{\rho}_e$ is the usual factor combining compressibility and heat transfer effects, and the correlation R_{uv} is, of course, the key mechanism to the shear stress. Using the present cold wall results as an example, the terms $\langle u'\rangle/\bar{u}_e$, $\langle v'\rangle/\bar{u}_e$ and $(\bar{\rho}/\bar{\rho}_e)$ are decreased by factors of 2, 8, and 2, respectively, from their low-speed values. These effects, which are in agreement with the trends demonstrated in Ref. 17, explain the 30-fold reduction observed in the measured value of the stress. In this connection, it should be pointed out that the $\langle v'\rangle$ measurements reported by Mikulla and Horstman[8] seem unusually high and, in fact, exceed the $\langle u'\rangle$ measurements made earlier in the same flow.[16] This is in contrast to the present findings and may account for the higher shear stress found in their experiment.

In summary then, the present measurements serve to bridge the gap between existing supersonic and hypersonic data. It is shown that at hypersonic speeds, the turbulent shear stress decreases rapidly with increasing Mach number and that this is directly a consequence of a corresponding reduction in the levels of the various flow fluctuations which contribute to the shear stress. Although a two-fold reduction in wall temperature is found to produce a similar effect on the turbulent stress, the data indicate clearly that the Mach number exerts a dominant influence on the shear mechanism in the hypersonic regime.

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